

Matrix Multiplication Order in Combined Transformations

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Let M_1, M_2, M_3, M_4 be transformation matrices applied in sequence to a local frame, with M_1 applied first and M_4 applied last. Here is a quick overview of how to multiply the individual transformation matrices to obtain the combined matrix for four relevant cases:

1. Successive transformations of a local frame about the global, static reference frame:

(a) Local→Global coordinate transformation:

Premultiplication of regular transformation matrices, e.g.:

$$P_{global} = M_4 * M_3 * M_2 * M_1 * P_{local}$$

(b) Global→Local coordinate transformation:

Postmultiplication of the inverse transformation matrices:

$$P_{local} = M_1^{-1} * M_2^{-1} * M_3^{-1} * M_4^{-1} * P_{global}$$

2. Successive transformations of a local frame about the local, moving frame axes:

(a) Local→Global coordinate transformation:

Postmultiplication of regular transformation matrices:

$$P_{global} = M_1 * M_2 * M_3 * M_4 * P_{local}$$

(b) Global→Local coordinate transformation:

Premultiplication of the inverse transformation matrices:

$$P_{local} = M_4^{-1} * M_3^{-1} * M_2^{-1} * M_1^{-1} * P_{global}$$

Notes: “Regular transformation matrix” refers to a column-major matrix where the elemental rotation matrices around the x and z axes have the minus sign in the upper right corner. Cases 1(a) and 2(b) correspond to our intuitive understanding of fixed and local axis transformations, respectively, while the other case are simply derived by multiplying each side with the inverse of the compound transformation matrix.